

# Third Family Corrections to Tri-bimaximal Lepton Mixing and a New Sum Rule

Stefan Antusch\*

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut) Föhringer Ring 6, D-80805 München, Germany

Stephen F. King† and Michal Malinský‡

School of Physics and Astronomy, University of Southampton, SO16 1BJ Southampton, United Kingdom

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We investigate the theoretical stability of the predictions of tri-bimaximal neutrino mixing with respect to third family wave-function corrections. Such third family wave-function corrections can arise from either the canonical normalisation of the kinetic terms or renormalisation group running effects. At leading order both sorts of corrections can be subsumed into a single universal parameter. For hierarchical neutrinos, this leads to a new testable lepton mixing sum rule  $s = r \cos \delta + \frac{2}{3}a$  (where  $s, r, a$  describe the deviations of solar, reactor and atmospheric mixing angles from their tri-bimaximal values, and  $\delta$  is the observable Dirac CP phase) which is stable under all leading order third family wave-function corrections, as well as Cabibbo-like charged lepton mixing effects.

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## I. INTRODUCTION

Since the discovery of neutrino masses and large lepton mixing angles, the flavour problem of Standard Model (SM) has received much attention. As the precision of the neutrino data has improved, it has become apparent that lepton mixing is consistent with the so called Tri-bimaximal (TB) mixing pattern [1],

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot P_M \quad (1)$$

where  $P_M$  is the so far experimentally undetermined diagonal phase matrix encoding the two observable Majorana phase differences. Many models attempt to reproduce this as a theoretical prediction [2, 3, 4, 5, 6, 7, 8, 9]. Since the forthcoming neutrino experiments will be sensitive to small deviations from TB mixing, it is important to quantify the “theoretical” uncertainty inherent in such TB mixing predictions.

In many classes of models TB mixing arises purely from the neutrino sector [10], subject to deviations due to charged lepton sector corrections [2, 3]. If these charged lepton corrections are “Cabibbo-like” in nature (i.e. dominated by a 1-2 mixing), it leads to a predictive sum rule [2] which may be expressed in terms of the parametrisation in [11] as  $s = r \cos \delta$ , where  $s$  and  $r$  describe the deviations of solar and reactor mixing angles from their tri-bimaximal values, and  $\delta$  is the observable Dirac CP phase in the standard parameterisation [12].

Another source of theoretical uncertainty in TB mixing schemes is the renormalisation group (RG) running [13]

of the relevant quantities from the high energy (usually the unification scale  $M_G$ ), where the theory is defined, to the electroweak scale  $M_Z$  appropriate for experimental measurements. The dominant source of RG corrections to lepton mixing arises typically from the large tau lepton and third family neutrino Yukawa couplings, leading to relatively large wave-function corrections in the framework of supersymmetric models. Such RG corrections can be readily estimated analytically [14, 15] for the TB mixing case with hierarchical light neutrinos considered here. Diagrammatically, such RG corrections correspond to loop diagrams involving third family matter and Higgs fields and their superpartners. Although suppressed by the loop factor of  $1/16\pi^2$ , they can be relevant since the loop factor is multiplied by a large logarithm of the ratio of energy scales.

Apart from RG effects there is another type of third family wave-function correction which emerges at tree-level in certain classes of models, and thus can potentially be rather large. These corrections modify the kinetic terms in the Lagrangian causing them to deviate from the standard (or canonical) form. Before the theory can be reliably interpreted, field transformations must be performed in order to return the kinetic terms back to canonical form which, however, leads to appropriate modifications of the Yukawa couplings. It is interesting that these effects are largest in many of the theories that predict TB mixing, especially those based on non-Abelian family symmetries spanning all three families of SM matter (see e.g. [16, 17, 18]). In such models the canonical normalisation (CN) corrections can in certain cases even exceed the effects due to RG running.

In this Letter we shall provide a unified treatment of all the above sources of theoretical corrections to the TB mixing, namely due to: *i*) RG corrections, *ii*) CN corrections and *iii*) charged lepton corrections. We will present a novel testable neutrino mixing sum rule which, at leading order, is stable under all these effects.

\*Electronic address: antusch@mppmu.mpg.de

†Electronic address: sfk@hep.phys.soton.ac.uk

‡Electronic address: malinsky@phys.soton.ac.uk

## II. GENERAL FORMALISM

Suppose the original (before the RG and CN corrections are accounted for) charged lepton ( $l$ ) and Majorana neutrino mass matrices  $\hat{M}_l$  and  $\hat{m}_\nu$  are diagonalised by means of unitary transformations  $\hat{V}_L^l \hat{M}_l \hat{V}_R^{l\dagger} = \hat{M}_l^D$  (we shall work in LR chirality basis) and  $\hat{V}_L^\nu \hat{m}_\nu \hat{V}_L^{\nu T} = \hat{m}_\nu^D$  so that the uncorrected lepton mixing matrix obeys  $\hat{U}_{\text{PMNS}} = \hat{V}_L^l \hat{V}_L^{\nu\dagger}$ .

The effect of both CN and leading logarithmic RG corrections on the  $\hat{M}_l$  and  $\hat{m}_\nu$  matrices can be described by a pair of transformation matrices  $P_{L,R}$

$$\hat{M}_l \rightarrow P_L^T \hat{M}_l P_R \equiv M_l, \quad \hat{m}_\nu \rightarrow P_L^T \hat{m}_\nu P_L \equiv m_\nu \quad (2)$$

which induce a relevant change on  $\hat{V}_{L,R}^l \rightarrow V_{L,R}^l$  and  $\hat{V}_L^\nu \rightarrow V_L^\nu$  so that  $V_L^l M_l V_R^{l\dagger} = M_l^D$  and  $V_L^\nu m_\nu V_L^{\nu T} = m_\nu^D$  and thus  $U_{\text{PMNS}} = V_L^l V_L^{\nu\dagger}$  is the *physical* lepton mixing matrix (after global rephasing). One can always write  $P_{L,R} = p_{L,R}(\mathbb{1} + \Delta P_{L,R})$  where, as we shall see, the constants  $p_{L,R}$  have no effect on the mixing angles and  $\Delta P_{L,R}$  denote the corrections from the flavour non-universal part of the RG and CN effects to be identified later. Equation (2) then implies

$$p_L p_R V_L^l (\mathbb{1} + \Delta P_L^T) \hat{M}_l (\mathbb{1} + \Delta P_R) V_R^{l\dagger} = M_l^D, \\ p_L^2 V_L^\nu (\mathbb{1} + \Delta P_L^T) \hat{m}_\nu (\mathbb{1} + \Delta P_L) V_L^{\nu T} = m_\nu^D. \quad (3)$$

If all the physical spectra are sufficiently hierarchical, the smallness of  $\Delta P_{L,R}$  factors ensures only small differences between  $\hat{V}_L^f$  and  $V_L^f$  (for  $f = l, \nu$ ), in particular

$$V_L^f = W_L^f \hat{V}_L^f = e^{i\Delta W_L^f} \hat{V}_L^f \quad (4)$$

where  $W_L^f$  are small unitary rotations in the unity neighborhood with  $\Delta W_L^f$  denoting their Hermitean generators. One can disentangle the left-handed and right-handed rotations in the charged lepton formula in (3) by considering  $M_l M_l^\dagger$  with the result [22]

$$(\Delta W_L^l)_{ij}^{i \neq j} \approx \frac{i}{\hat{m}_j^{l2} - \hat{m}_i^{l2}} \left[ (\hat{m}_i^{l2} + \hat{m}_j^{l2}) (\hat{V}_L^l \Delta P_L^T \hat{V}_L^{l\dagger})_{ij} \right. \\ \left. + 2\hat{m}_i^l \hat{m}_j^l (\hat{V}_R^\nu \Delta P_R \hat{V}_R^{l\dagger})_{ij} \right], \quad (5)$$

where the eigenvalues  $\hat{m}_i^{l2}$  of the original  $\hat{M}_l$  matrix can, at leading order, be identified with the physical charged lepton masses. Similarly, the neutrino sector corrections obey (replacing  $\hat{M}_l \rightarrow \hat{m}_\nu$ ,  $V_L^l \rightarrow V_L^\nu$ ,  $V_R^l \rightarrow V_L^{\nu*}$  and  $\Delta P_R \rightarrow \Delta P_L$  in formula (5) above)

$$(\Delta W_L^\nu)_{ij}^{i \neq j} \approx \frac{i}{\hat{m}_j^{\nu 2} - \hat{m}_i^{\nu 2}} \left[ (\hat{m}_i^{\nu 2} + \hat{m}_j^{\nu 2}) (\hat{V}_L^\nu \Delta P_L^T \hat{V}_L^{\nu\dagger})_{ij} \right. \\ \left. + 2\hat{m}_i^\nu \hat{m}_j^\nu (\hat{V}_L^\nu \Delta P_L^T \hat{V}_L^{\nu\dagger})_{ji} \right]. \quad (6)$$

From equations (4), (5) and (6) one can write the corrected (i.e. *physical*) lepton mixing matrix  $U_{\text{PMNS}} = V_L^l V_L^{\nu\dagger}$  in terms of the original  $\hat{U}_{\text{PMNS}}$  as  $U_{\text{PMNS}} = \hat{U}_{\text{PMNS}} + \Delta U_{\text{PMNS}}$  where

$$\Delta U_{\text{PMNS}} \approx i \left( \Delta W_L^l \hat{U}_{\text{PMNS}} - \hat{U}_{\text{PMNS}} \Delta W_L^{\nu\dagger} \right). \quad (7)$$

Due to the assumed hierarchy in the physical spectra, the first terms in equations (5) and (6) dominate over the second (thus avoiding the ambiguity in the unknown structure of the right-handed (RH) rotations in the charged lepton sector) and so we shall neglect the latter and focus on the left-handed (LH) sector.

The RG effects in the supersymmetric case yield at leading order [13]  $P_L^{RG} = r_L \mathbb{1} + \Delta P_L^{RG}$  where

$$r_L = 1 - \frac{1}{16\pi^2} \left[ 3(\text{Tr } Y_u^\dagger Y_u - \hat{g}^2) \ln \frac{M_G}{M_Z} + \text{Tr } Y_\nu^\dagger Y_\nu \ln \frac{M_G}{M_N} \right] \quad (8)$$

(with  $\hat{g}^2 \equiv g_2^2 + \frac{1}{5}g_1^2$ ) accounts for flavour-universal contribution while

$$\Delta P_L^{RG} = -\frac{1}{16\pi^2} \left[ Y_l^* Y_l^T \ln \frac{M_G}{M_Z} + Y_\nu^* Y_\nu^T \ln \frac{M_G}{M_N} \right], \quad (9)$$

denotes the flavour non-trivial piece. In (8) and (9),  $M_N$  denotes the mass of the lightest RH neutrino. Notice that since  $r_L$  is close to 1 one can write at leading order

$$P_L^{RG} \approx r_L (\mathbb{1} + \Delta P_L^{RG}) \quad (10)$$

rendering the  $r_L$  factor irrelevant for the mixing angles.

Turning to CN effects, with the non-canonical LH lepton doublet ( $L$ ) kinetic term written as  $iL^\dagger \not{D} K_L L$  and  $K_L = k_L(\mathbb{1} + \Delta K_L)$ , the CN transformation can be written in the form of (2) with (c.f. [16, 17, 18])

$$P_L^{CN} = (\mathbb{1} + \Delta P_L^{CN}) / \sqrt{k_L} \quad \text{where} \quad \Delta P_L^{CN} \approx -\frac{1}{2} \Delta K_L. \quad (11)$$

At leading order,  $P_L^{CN}$  fulfills  $(P_L^{CN})^{-1\dagger} (P_L^{CN})^{-1} = K_L$ .

Finally, one can combine both RG and CN effects under a single transformation satisfying at leading order

$$P_L \approx P_L^{CN} P_L^{RG} \approx \frac{r_L}{\sqrt{k_L}} (\mathbb{1} + \Delta P_L^{CN} + \Delta P_L^{RG}) \quad (12)$$

yielding the assumed form of  $P_L = p_L(\mathbb{1} + \Delta P_L)$  with  $p_L = r_L / \sqrt{k_L}$  and  $\Delta P_L = \Delta P_L^{RG} + \Delta P_L^{CN}$ .

Using these results, the leading order corrections to lepton mixing from RG and CN effects can be calculated.

## III. THIRD FAMILY CORRECTIONS TO TRI-BIMAXIMAL LEPTON MIXING

In this section we shall apply the formalism of section II to TB lepton mixing, where we assume to begin with that this mixing originates entirely from the neutrino sector and subsequently extend the analysis to include corrections from charged lepton mixing [10]. Thus, let us

assume first that the lepton mixing predicted by some underlying theory in the absence of RG and CN corrections happens to be exactly tri-bimaximal  $\hat{U}_{\text{PMNS}} = \hat{V}_L^l \hat{V}_L^{\nu\dagger} = U_{\text{TB}}$  where  $\hat{V}_L^l = \mathbb{1}$  while  $\hat{V}_L^{\nu\dagger} = U_{\text{TB}}$ . Including RG and CN corrections,  $\hat{V}_L^l$  and  $\hat{V}_L^{\nu\dagger}$  then change according to (4) with the correction matrices given by equations (5), (6).

Here we shall restrict ourselves to the dominant third family wavefunction corrections, so that from (9) one can write  $\Delta P_L^{RG} = -\frac{1}{2}\text{diag}(0, 0, \eta^{RG})$  with  $\eta^{RG} = [y_\tau^2 \ln(M_G/M_Z) + y_{\nu 3}^2 \ln(M_G/M_N)]/8\pi^2$ . If the third family effects dominate also the form of  $K_L$  in equation (11),  $\Delta K_L$  is a matrix controlled by the 33 entry and  $\Delta P_L^{CN} = -\frac{1}{2}\text{diag}(0, 0, \eta^{CN})$  with  $\eta^{CN} = (\Delta K_L)_{33}$ . Then at the leading order  $\Delta P_L = -\frac{1}{2}\text{diag}(0, 0, \eta)$  is governed by a *universal* parameter  $\eta = \eta^{RG} + \eta^{CN}$ . This is the case, for instance, in all models in which a family symmetry spans all three SM matter families [16, 17, 18]. In such theories the third family corrections to the kinetic function  $K_L$  have the same origin as the third family Yukawa couplings, and  $\eta^{CN}$  can be as large as  $y_\tau^2$  (in the  $SO(3)$ -type of models) or  $y_\tau$  (for underlying  $SU(3)$  flavour symmetry). However, it is also possible that  $\eta^{CN} \ll y_\tau$  in certain classes of models, and the effect is strongly dependent on the details of the underlying theory [6, 20].

Given the diagonal form of  $\Delta P_L$ , and our assumption that  $\hat{V}_L^l = \mathbb{1}$ , one gets from equation (5)  $\Delta W_L^l = 0$  and thus  $V_L^l = \mathbb{1}$  so the corrected lepton mixing matrix is given by just  $V_L^{\nu\dagger} = U_{\text{TB}} + \Delta U_{\text{TB}}$  with  $\Delta U_{\text{TB}} = -iU_{\text{TB}}\Delta W_L^{\nu\dagger}$  from equation (7). If the neutrino spectrum is hierarchical, the leading (first) term in (6) yields

$$\Delta(U_{\text{TB}})_{ij}^{i<j} \approx -(U_{\text{TB}})_{ij} \left( U_{\text{TB}}^\dagger \Delta P_L^T U_{\text{TB}} \right)_{jj} \quad (\text{no } j \text{ sum}). \quad (13)$$

We can see that, at the leading order, the corrections to  $(U_{\text{TB}})_{ij}$  for  $i < j$  are proportional to  $(U_{\text{TB}})_{ij}$  itself, and thus for example both RG and CN corrections to  $\theta_{13}$  are zero due to  $(U_{\text{TB}})_{13} = 0$  [23]. However the result  $\Delta(U_{\text{TB}})_{13} = 0$  arises only at leading order in small quantities ( $\eta$  and  $\sqrt{\Delta m_{21}^2/\Delta m_{31}^2} \approx m_2/m_3 \approx 1/5$ ) and gets lifted at next-to-leading level. Restoring the second term in equation (6) one obtains:

$$\theta_{13} \approx \frac{1}{3\sqrt{2}}|\eta| \sqrt{\Delta m_{21}^2/\Delta m_{31}^2} \approx 4 \times 10^{-2}|\eta|. \quad (14)$$

All together, this yields at the leading order

$$V_L^{\nu\dagger} \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{1}{12}\eta) & \frac{1}{\sqrt{3}}(1 + \frac{1}{6}\eta) & 0.04 \times |\eta| e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 - \frac{1}{12}\eta) & \frac{1}{\sqrt{3}}(1 - \frac{1}{3}\eta) & \frac{1}{\sqrt{2}}(1 + \frac{1}{4}\eta) \\ \frac{1}{\sqrt{6}}(1 + \frac{5}{12}\eta) & -\frac{1}{\sqrt{3}}(1 + \frac{1}{6}\eta) & \frac{1}{\sqrt{2}}(1 - \frac{1}{4}\eta) \end{pmatrix} \quad (15)$$

which is unitary up to  $\mathcal{O}(\eta^2)$ . In terms of the deviations from the exact TB mixing parametrized [11] by  $\sin\theta_{12} = (1 + s)/\sqrt{3}$ ,  $\sin\theta_{23} = (1 + a)/\sqrt{2}$  and  $\sin\theta_{13} = r/\sqrt{2}$  one gets (so far without including charged

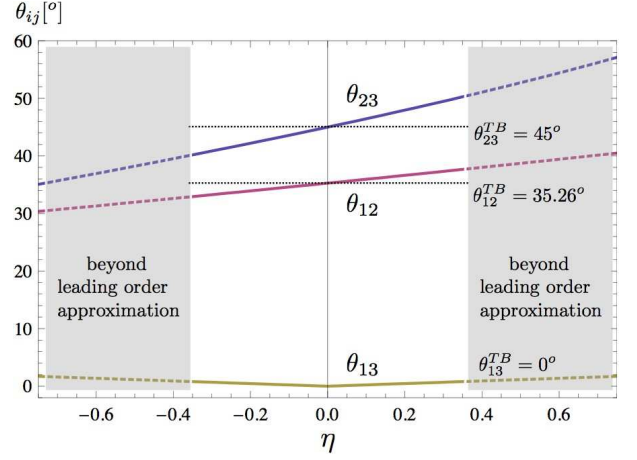


FIG. 1: Renormalisation group and canonical normalization corrections to tri-bimaximal neutrino mixing. The shaded regions correspond to  $\eta$ -values outside the linear approximation.

lepton corrections):

$$r \approx 6 \times 10^{-2}|\eta|, \quad s = \frac{1}{6}\eta \quad \text{and} \quad a = \frac{1}{4}\eta. \quad (16)$$

We see that in particular  $\theta_{13}$  is rather stable and the atmospheric  $\theta_{23}$  is changing faster with  $\eta$  than the solar  $\theta_{12}$ , as shown in FIG. 1.

In any realistic model, the charged lepton mixing corrections entering  $U_{\text{PMNS}}$  must be taken into account. It is well known that if  $\hat{V}_L^l$  is Cabibbo-like with  $\theta_{12}^l$  being the only non-negligible mixing angle, then (ignoring RG and CN effects) this gives rise to a particular pattern of corrections to  $\theta_{13}$  and  $\theta_{12}$  that obey (at the high scale) the relation  $s = r \cos\delta$  with, e.g.,  $r \approx \theta_C/3$  for  $\theta_{12}^l \approx \theta_C/3$  ( $\theta_C$  is the Cabibbo angle) in many unified models [2].

We can include the leading order RG and CN corrections to  $s = r \cos\delta$  by considering only the neutrino sector effects (encoded in  $\Delta W_L^\nu$ ) for the individual terms. (This follows since for the Cabibbo-like  $\hat{V}_L^l$  and  $\Delta P_L$  dominated by the 33 entry, it is still the case that  $\Delta W_L^l = 0$ .) Therefore the previous example provides a good estimate of the relevant corrections at leading order in small quantities (including now also the charged lepton mixing  $\theta_{12}^l$ ). Neglecting the subleading correction in (14), from equation (16) one obtains  $s = r \cos\delta + \frac{1}{6}\eta$ , which can be rewritten in terms of only measurable quantities in form of a new sum rule

$$s = r \cos\delta + \frac{2}{3}a. \quad (17)$$

The new sum rule in equation (17) is stable under the considered theoretical corrections and additionally involves the deviation of atmospheric mixing from maximality [19]. The main sources of remaining uncertainties in formula (17) are the neglected (order  $4\%|\eta|$ ) corrections to  $r \cos\delta$  due to the subleading contribution (14), the higher order corrections to  $\Delta P_L^{RG}$  and  $\Delta P_L^{CN}$  (all suppressed by the relevant Yukawa coupling ratios) and

the higher order  $\eta$ -effects. A rigorous derivation and detailed discussion of formula (17) will be given in a forthcoming paper [20].

#### IV. CONCLUSIONS

We have presented a unified formalism for dealing with both renormalisation group running effects and canonical normalisation corrections. Using this formalism we have investigated the third family wave-function corrections to the theoretical predictions of tri-bimaximal neutrino mixing. We found that at leading order both effects can be subsumed into a single universal parameter  $\eta$ . Including also the leading order Cabibbo-like charged lep-

ton mixing corrections, which typically arise in unified flavour models, we have derived the theoretically stable sum rule  $s = r \cos \delta + \frac{2}{3}a$  where  $s$ ,  $r$  and  $a$  parametrize the deviations of the solar, reactor and atmospheric mixing angles from their tri-bimaximal values and  $\delta$  is the leptonic Dirac CP phase. Such a sum rule is testable in future high precision neutrino experiments [21].

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strained so we shall conventionally put  $(\Delta W_L^{l,\nu})_{ii} = 0$ .  
 [23] Note also that (13) implies that the Majorana phases

have no effect on the corrections to the mixings angles.